Quiz 3 #3:

$$f(x,y) = ye^{xy}$$

Tangent plane @ $(x,y) = (0,1)$.

$$f_y(x,y) = e^{xy} + ye^{xy}x$$

(Common mistake was computing f_y incorrectly)

You might remember the tan. plane eqn as

$$z = z_0 + \underbrace{f_x}_{\substack{\text{supposed to be} \\ f_x(x_0,y_0)}} \cdot (x - x_0) + f_y \cdot (y - y_0)$$

ex) $z = x^2 + y^2$ — x, y, z are real #s and this eqn describes a relationship between them.

$f(x,y) = x^2 + y^2$ — f is a function $\mathbb{R}^2 \longrightarrow \mathbb{R}$ and this equation describes what f does to an arbitrary input

Note: $f(u,v) = u^2 + v^2$ defines the exact same function f.

Common mistake: using $f_x(x,y)$ instead of $f_x(x_0,y_0)$ in the tangent plane equation, and writing

$$z = 1 + y^2 e^{xy}(x-0) + (e^{xy} + xye^{xy})(y-1)$$

(But this is most definitely not even a plane equation, plus it looks way harder to work with than the original $f(x,y)$!)

Two equivalent ways of writing tan plane eqn:

$$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0 \qquad (*)$$

$$\nabla F(\vec{r}_0) \cdot (\vec{r} - \vec{r}_0) = 0$$

Mistake I saw: People used (*) with $F(x,y,z) = ye^{xy}$.

Corrected ver: write

$$z = ye^{xy}$$

rearrange to $ye^{xy} - z = 0$

this is $F(x,y,z)$

#1) $$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy+x^2}$$

Try along lines $y=mx$

$$\lim_{x\to 0} \frac{x^2+m^2x^2}{x^2m+x^2}$$

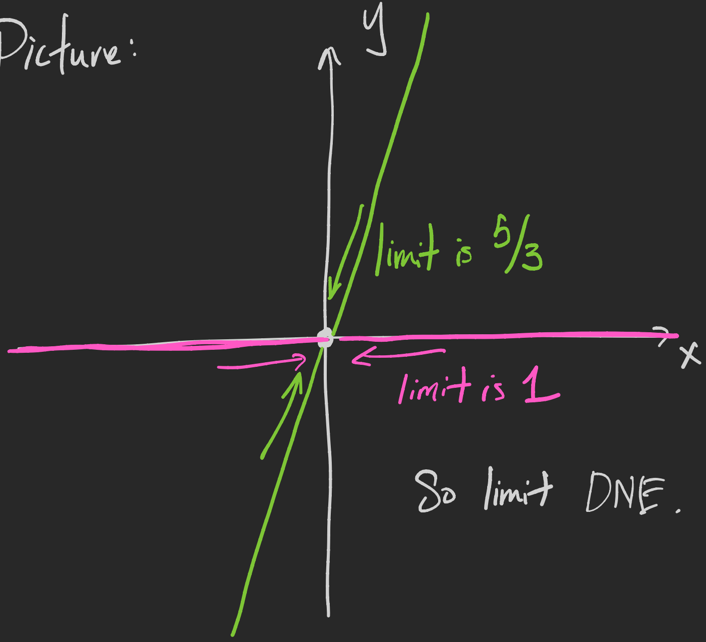
$$= \lim_{x\to 0} \frac{1+m^2}{m+1} = \frac{1+m^2}{m+1}$$

This means, along $y=0$ (the x-axis) the limit is 1

but along $y=2x$ the limit is $5/3$.

So the limit DNE.

Picture:



So limit DNE.

Worksheet #2(a)

want directional deriv of $f(x,y)$ at this point in this direction